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HYDRODYNAMIC ANALYSIS OF THE PROCESS OF MAKING THREE-LAYER OPTICAL FIBERS AND CALCULATION OF THE FIELD OF ELASTIC STRESSES AND BIREFRINGENCE

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One of the methods used to make semifinished products for the manufacture of polarization-maintaining optical fibers is based on the use of the surface tension of glass in the liquid state [1]. The initial cross section of the semifinished product is shown in Fig. 1, where the region 0 corresponds to the core through which the signal propagates. The numbers 1 and 2 denote the straining and cladding sheaths, which are designed to create a stress state in the core. Part of the clad is removed — as shown by the dashed lines in Fig. 1, for example — and the semifinished product is placed in a furnace and heated. During heating, the straining and cladding sheaths become liquid and the surface tension at the boundary Γ_2 begins to round it off. The resulting flow of molten glass deforms the boundary Γ_1, which is subjected to a low surface tension. The deformation of this boundary causes it to lose its circular form. Meanwhile, the core remains solid and boundary Γ_0 remains unchanged. After completion of the process of rounding-off of the boundary Γ_2, a semifinished product with a noncircular boundary Γ_1 is obtained. Due to the difference in the thermoelastic properties of the materials in the straining and cladding layers of the semifinished product (and optical fiber), an anisotropic field of elastic stresses is created along with the associated birefringence. Accordingly, the core becomes capable of transmitting signals with a certain polarization.

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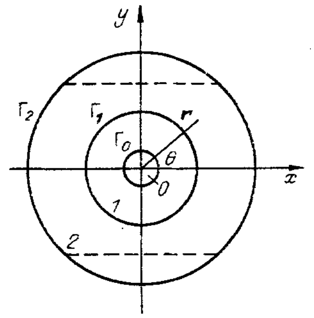

Fig. 1

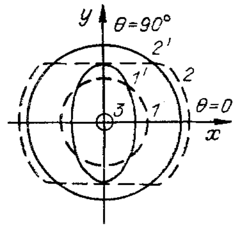

Fig. 2

The authors of [2, 3] calculated the flow process in the roundoff of the external boundary of a two-layer semifinished product (in an approximation which ignored the effect of the core 0). The flow was assumed to have been a noninertial Stokes flow in these studies. A comparison with experimental data was made in [3]. Here, we generalize the solution in [2, 3] to the case of a three-layer semifinished product. As was shown in [2, 3], the structure of the semifinished product remains nearly the same in the fiber ultimately formed from it. Thus, the results of the solution of the hydrodynamic problem were used to calculate the elastic stresses and birefringence in the optical fiber. Several analytical solutions to the elastic problem were obtained in [4, 5] for fibers having a cross section with a different structure.

1. The hydrodynamic problem is solved with the use of the Stokes equations [6, 7], which converge to a biharmonic equation for the stream function ψ^*:

$$\Delta\Delta\psi^* = 0 \tag{1.1}$$

(here and below, Δ is the two-dimensional Laplace operator).

Having constructed the general solution of Eq. (1.1) in the form of a Fourier series, we use a stream function ψ^* which is periodic with respect to θ to find the fields of velocity, pressure, and viscous stresses in regions 1 and 2. Some details of these calculations were presented in [2]. As in [2, 3], the boundary conditions of the hydrodynamic problem will be linearized here. We will assume that the boundaries Γ_1 and Γ_2 are small perturbations of circles of radii R_1 and R_2. This approach retains all of the boundary conditions (11) from [2] but, in contrast to [2], employs the condition of adhesion at $r = R_0$ (the boundary of the core and sheath 1) rather than the condition of finiteness of the solution at $r = 0$

$$v_{r1} = 0, \; v_{\theta 1} = 0 \tag{1.2}$$

(v_r and v_θ are components of velocity in polar coordinates: the additional subscript 1 corresponds to region 1).

With the satisfaction of Eq. (1.2), $\psi_1^* \equiv 0$ on the boundary Γ_0 ($r = R_0$). Conditions (1.2) imply that even in the case of nonsymmetric removal of part of sheath 2, the solid core is not rotated by viscous stresses during the roundoff of boundary Γ_2 because the semifinished product is compressed at its two solid ends (at $z = \pm L/2$, where z is the axial coordinate and L is the length of the semifinished product).

Having constructed the general solution of the hydrodynamic problem and having satisfied boundary conditions (1.2) and (11) from [2], we obtain expressions for the coefficients of Fourier series which describe the perturbations of boundaries Γ_1 and Γ_2 during the flow process:

$$\zeta_i = \frac{b_{0i}(t)}{2} + \sum_{n=1}^{\infty} [a_{ni}(t)\sin n\theta + b_{ni}(t)\cos n\theta] \quad (i = 1, 2) \tag{1.3}$$

(the boundaries correspond to $r_i = R_i(1 + \zeta_i)$).

The expressions for the coefficients a_{ni} and b_{ni} will be (14), (22), and (24)-(28) from [2], with the replacment of S_1, S_2, S_7, and S_8 by

$$S_1 = \frac{\mu_1}{\mu_2}\{(-2n - 4 + 2n^2) + [n(-2n^2 - 2n)\gamma_0^{2n+2} - (n+1)(-2n + 4 - 2n^2)\gamma_0^{2n}]\},$$

$$S_2=\frac{\mu_1}{\mu_2}\{(2n^2-2n)+[(n-1)(-2n^2-2n)\gamma_0^{2n}-n(-2n+4-2n^2)\gamma_0^{2n-2}]\},$$
$$S_7=\frac{\mu_1}{\mu_2}\{(-2n^2-2n)+[n(-2n^2-2n)\gamma_0^{2n+2}-(n+1)(-2n^2+2n)\gamma_0^{2n}]\},\tag{1.4}$$
$$S_8=\frac{\mu_1}{\mu_2}\{(-2n^2+2n)+[(n-1)(-2n^2-2n)\gamma_0^{2n}-n(-2n^2+2n)\gamma_0^{2n-2}]\}\quad\left(\gamma_0=\frac{R_0}{R_1}\right),$$

and the addition of S_{15}-S_{18}:

$$S_{15}=(n+2)+[-n^2\gamma_0^{2n+2}-(n+1)(-n+2)\gamma_0^{2n}],$$
$$S_{16}=n+[-n(n-1)\gamma_0^{2n}-n(-n+2)\gamma_0^{2n-2}],\tag{1.5}$$
$$S_{17}=1+[n\gamma_0^{2n+2}-(n+1)\gamma_0^{2n}],\ S_{18}=1+[(n-1)\gamma_0^{2n}-n\gamma_0^{2n-2}]$$

as well as the replacement of k_1-k_8 by

$$k_1=(S_{15}S_{18}-S_{16}S_{17})(S_3S_8-S_2S_9)-(S_{18}S_{12}-(S_{16}/n))(S_1S_8-S_2S_7),$$
$$k_2=(S_{15}S_{18}-S_{16}S_{17})(S_4S_8-S_2S_{10})-S_{18}S_{13}(S_1S_8-S_2S_7),$$
$$k_3=(S_{15}S_{18}-S_{16}S_{17})(S_5S_8-S_2S_{11})-S_{18}S_{14}(S_1S_8-S_2S_7),$$
$$k_4=(S_{15}S_{18}-S_{16}S_{17})S_6S_8,\tag{1.6}$$
$$k_5=(S_{15}S_{18}-S_{16}S_{17})(S_1S_9-S_3S_7)-((S_{15}/n)-S_{17}S_{12})(S_1S_8-S_2S_7),$$
$$k_6=(S_{15}S_{18}-S_{16}S_{17})(S_1S_{10}-S_4S_7)+S_{17}S_{13}(S_1S_8-S_2S_7),$$
$$k_7=(S_{15}S_{18}-S_{16}S_{17})(S_1S_{11}-S_5S_7)+S_{17}S_{14}(S_1S_8-S_2S_7),$$
$$k_8=-(S_{15}S_{18}-S_{16}S_{17})S_6S_7.$$

In Eq. (1.4), μ_1 and μ_2 are the viscosities of the molten glass in sheaths 1 and 2. At $\gamma_0=0$, Eqs. (1.4)-(1.6) take the form of the results in [2, 3].

2. After the formation of the internal structure of a semifinished product with a noncircular boundary Γ_1 has been completed, it is necessary to address the problem of the distribution of the elastic stresses in the semifinished product (or fiber). In the solution of this problem, we make use of the coefficients of the Fourier series for Γ_1, corresponding to the moment of cessation of flow. The values of a_{n1*} and b_{n2*} are known from the solution of the hydrodynamic problem; $a_{n2*}\approx 0$, $b_{n1*}\approx 0$ $(n\geq 2)$. The elastic deformation is assumed to be planar.

In accordance with [8], calculation of the elastic stresses reduces to finding the potential $\chi=\varphi+\psi$, the sum of the thermoelastic potential φ and the elastic potential ψ. These quantities satisfy the equations

$$\Delta\varphi=\frac{1+\nu}{1-\nu}\alpha_l(T-T_0),\ \Delta\Delta\psi=0,\tag{2.1}$$

$$\sigma_{rr}=-\frac{E}{1+\nu}\left(\frac{1}{r}\frac{\partial\chi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\chi}{\partial\theta^2}\right),\ \sigma_{r\theta}=\frac{E}{1+\nu}\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial\chi}{\partial\theta}\right),\ \sigma_{\theta\theta}=-\frac{E}{1+\nu}\frac{\partial^2\chi}{\partial r^2}.$$

Here, ν is the Poisson's ratio, which (together with the Young's modulus E) is henceforth assumed to be identical for the materials in regions 0-2; α_ℓ is the coefficient of linear thermal expansion, which is different in regions 0-2; T is temperature (assumed to be constant over the cross section); T_0 is the temperature at which thermoelastic stresses are absent. In general, this temperature differs in regions 0-2. We will also use the notation $\beta_i=[(1+\nu)/(1-\nu)]\cdot\alpha_{\ell i}(T-T_{0i})$ $(i=0,1,2)$.

The thermoelastic displacements $\mathbf{u}_t=\nabla\varphi$ are connected with the thermoelastic potential. They should be continuous on the internal boundaries of the regions, since the displacements connected with ψ are automatically continuous. Thus, in an approximation which is linear with respect to the perturbations, the solution of the first equation of (2.1) must satisfy the conditions

$$r=R_0:\ \partial\varphi_0/\partial r=\partial\varphi_1/\partial r,\ \partial\varphi_0/\partial\theta=\partial\varphi_1/\partial\theta;$$
$$r=R_1(1+\zeta_{1*}):\ \partial\varphi_1/\partial r=\partial\varphi_2/\partial r,\ \partial\varphi_1/\partial\theta=\partial\varphi_2/\partial\theta.$$

Here, the linearized conditions for the thermoelastic stresses, connected with φ and denoted by the superscript t,

$$r=R_0:\ \sigma_{rr0}^t=\sigma_{rr1}^t,\ \sigma_{r\theta0}^t=\sigma_{r\theta1}^t;$$
$$r=R_1(1+\zeta_{1*}):\ \sigma_{rr1}^t=\sigma_{rr2}^t,\ \sigma_{r\theta1}^t+(\sigma_{rr1}^t-\sigma_{\theta\theta1}^t)\frac{\partial\zeta_1}{\partial\theta}=\sigma_{r\theta2}^t+(\sigma_{rr2}^t-\sigma_{\theta\theta2}^t)\frac{\partial\zeta_1}{\partial\theta}$$

are satisfied automatically, while the thermoelastic stresses connected with ψ are also continuous. It should be noted that the thermoelastic displacements are negligibly small even compared to ζ_1*, although the stresses connected with them may turn out to be substantial because they contain the multiplier E.

The stress σ_{rr} should be finite at r = 0, while on the circular external surface of the semifinished product (or fiber)

$$r \simeq R_2\colon \sigma_{rr} = \sigma_{r\theta} = 0.$$

The stress field calculated by this approach has the form

$$\begin{gathered}\sigma_{rr} = -\frac{E}{1+\nu}\Bigg[\frac{\beta_i}{2} + \frac{C_i}{r^2} + 2W + \sum_{n=1}^{\infty}\{[Q_n(n+2-n^2)r^n + \\ + T_n(n-n^2)r^{n-2} - T_{2n}(n+n^2)r^{-n-2}]\sin n\theta + [V_n(n+2-n^2)r^n + \\ + Y_n(n-n^2)r^{n-2} - Y_{2n}(n+n^2)r^{-n-2}]\cos n\theta\}\Bigg], \\ \sigma_{r\theta} = \frac{E}{1+\nu}\sum_{n=1}^{\infty}\{[Q_n(n^2+n)r^n + T_n(n^2-n)r^{n-2} - T_{2n}(n^2+n)r^{-n-2}]\times \\ \times\cos n\theta + [V_n(-n^2-n)r^n + Y_n(-n^2+n)r^{n-2} + (n^2+n)Y_{2n}r^{-n-2}]\sin n\theta\}, \\ \sigma_{\theta\theta} = -\frac{E}{1+\nu}\Bigg[\frac{\beta_i}{2} - \frac{C_i}{r^2} + 2W + \sum_{n=1}^{\infty}\{[Q_n(n+2)(n+1)r^n + n(n-1)T_nr^{n-2} + n(n+1)T_{2n}r^{-n-2}]\sin n\theta + \\ + [V_n(n+2)(n+1)r^n + n(n-1)Y_nr^{n-2} + n(n+1)Y_{2n}r^{-n-2}]\cos n\theta\}\Bigg].\end{gathered} \tag{2.2}$$

Here, we took the following in the calculations:

$$C_i = \begin{cases} 0, & i = 0, \\ \dfrac{R_0^2}{2}(\beta_0 - \beta_1), & i = 1, \\ \dfrac{R_0^2}{2}(\beta_0 - \beta_1) + \dfrac{R_1^2}{2}(\beta_1 - \beta_2), & i = 2; \end{cases} \tag{2.3}$$

$$W = -\frac{\beta_2}{4} - \frac{R_0^2}{4R_2^2}(\beta_0 - \beta_1) - \frac{R_1^2}{4R_2^2}(\beta_1 - \beta_2);\ Q_n = -(\beta_1 - \beta_2)R_1R_2^{-n-1}\frac{1}{2\gamma^{n+1}}a_{n1*};$$

$$V_n = -(\beta_1 - \beta_2)R_1R_2^{-n-1}\frac{1}{2\gamma^{n+1}}b_{n1*};\ T_1,\ Y_1 < \infty,\ \forall i;$$

$$\underset{(n\geqslant 2)}{T_n} = \begin{cases} \dfrac{(\beta_1-\beta_2)a_{n1*}R_1^{-n+2}}{2n}[(n+1)\gamma^{-2n} - 1], & i = 0, \\ \dfrac{(\beta_1-\beta_2)a_{n1*}R_1^{-n+2}}{2n}[(n+1)\gamma^{-2n} - 1], & i = 1, \\ \dfrac{(\beta_1-\beta_2)a_{n1*}(n+1)R_1R_2^{-n+1}}{2n\gamma^{n+1}}, & i = 2; \end{cases}$$

$$\underset{(n\geqslant 2)}{Y_n} = \begin{cases} \dfrac{(\beta_1-\beta_2)b_{n1*}R_1^{-n+2}}{2n}[(n+1)\gamma^{-2n} - 1], & i = 0, \\ \dfrac{(\beta_1-\beta_2)b_{n1*}R_1^{-n+2}}{2n}[(n+1)\gamma^{-2n} - 1], & i = 1, \\ \dfrac{(\beta_1-\beta_2)b_{n1*}(n+1)R_1R_2^{-n+1}}{2n\gamma^{n+1}}, & i = 2; \end{cases}$$

$$T_{2n} = \begin{cases} 0, & i = 0, \\ 0, & i = 1, \\ -\dfrac{(\beta_1-\beta_2)R_1^{n+2}}{2n}a_{n1*}, & i = 2; \end{cases} \qquad Y_{2n} = \begin{cases} 0, & i = 0, \\ 0, & i = 1, \\ -\dfrac{(\beta_1-\beta_2)R_1^{n+2}}{2n}b_{n1*}, & i = 2, \end{cases}$$

$\gamma = R_2/R_1$. In accordance with [5], the coefficient of birefringence

$$B = C(\sigma_{xx} - \sigma_{yy}) = C[(\sigma_{rr} - \sigma_{\theta\theta})\cos 2\theta - 2\sigma_{r\theta}\sin 2\theta] \tag{2.4}$$

(C is a proportionality factor whose value is known). Thus, with allowance for Eqs. (2.2) and (2.4), we find in the core (i = 0) that

$$B(r,\theta)=\frac{2CE}{1+\nu}\sum_{n=1}^{\infty}\{[Q_n(n^2+n)r^n+T_n(n^2-n)r^{n-2}]\sin[(n-2)\theta]+[V_n(n^2+n)r^n+Y_n(n^2-n)r^{n-2}]\cos[(n-2)\theta]\}.$$

In the center of the core at r = 0, $B = B_*$:

$$B_*=\frac{CE}{1+\nu}(\beta_2-\beta_1)\left(1-3\gamma^{-4}\right)b_{21*}. \tag{2.5}$$

As can be seen, in a linear approximation the value of B_* is determined by a single coefficient of Fourier series (1.3). This coefficient determines the harmonic $\cos 2\theta$ and changes sign at $\gamma = 3^{1/4}$. This finding is consistent with the results obtained in [4].

On the boundary of the core at $r = R_0 - 0$

$$B_0(\theta)=\frac{B(R_0-0,\theta)}{[2CE(\beta_1-\beta_2)/(1+\nu)]}=\sum_{n=1}^{\infty}\left\{-\frac{\gamma_0^n(n^2+n)}{2\gamma^{2n+2}}+\right. \tag{2.6}$$
$$\left.+\frac{1}{2}\gamma_0^{n-2}\left[(n+1)\gamma^{-2n}-1\right](n-1)\right\}\{a_{n1*}\sin[(n-2)\theta]+b_{n1*}\cos[(n-2)\theta]\}.$$

We can use Eqs. (2.2)-(2.4) to obtain a general expression for the dimensionless coefficient of birefringence in the cross section of the optical fiber. The expression has the form

$$B'=\frac{B(\rho,\theta)}{[2CE(\beta_1-\beta_2)/(1+\nu)]}=\sum_{n=1}^{\infty}\left\{-\frac{(n^2+n)}{2\gamma^{2n+2}}\rho^n+\frac{1}{2}\left[(n+1)\gamma^{-2n}-\right.\right.$$
$$\left.\left.-\omega\right](n-1)\rho^{n-2}\right\}\{a_{n1*}\sin[(n-2)\theta]+b_{n1*}\cos[(n-2)\theta]\}-\frac{\xi\cos 2\theta}{\rho^2}- \tag{2.7}$$
$$-\sum_{n=1}^{\infty}\frac{\omega_1}{2}(n+1)\rho^{-n-2}\{a_{n1*}\sin[(n+2)\theta]+b_{n1*}\cos[(n+2)\theta]\};\quad \rho=\frac{r}{R_1};$$

$$\omega=\begin{cases}1, & 0\leqslant\rho<1+\zeta_{1*}(\theta),\\ 0, & 1+\zeta_{1*}(\theta)<\rho<\gamma[1+\zeta_{2*}(\theta)];\end{cases}$$

$$\omega_1=\begin{cases}0, & 0\leqslant\rho<1+\zeta_{1*}(\theta),\\ 1, & 1+\zeta_{1*}(\theta)<\rho<\gamma[1+\zeta_{2*}(\theta)];\end{cases}$$

$$\xi=\begin{cases}0, & 0\leqslant\rho<\gamma_0,\\ \dfrac{\gamma_0^2}{2}\left(\dfrac{\beta_0-\beta_1}{\beta_1-\beta_2}\right), & \gamma_0<\rho<1+\zeta_{1*}(\theta),\\ \dfrac{\gamma_0^2}{2}\left(\dfrac{\beta_0-\beta_1}{\beta_1-\beta_2}\right)+\dfrac{1}{2}, & 1+\zeta_{1*}(\theta)<\rho<\gamma[1+\zeta_{2*}(\theta)].\end{cases}$$

Here, ζ_{1*} and ζ_{2*} are the perturbations of the boundaries Γ_1 and Γ_2 corresponding to the amount of cessation of the flow. These perturbations are known from the solution of the hydrodynamic problem.

Equations (2.5) and (2.6) can be obtained from Eq. (2.7) with $\rho = 0$ and $\rho = \gamma_0 - 0$. It should also be noted that the expressions for the stresses in the cross section (2.2)-(2.3) can be changed to the following form:

$$\sigma'_{rr}=\frac{\sigma_{rr}(\rho,\theta)}{[E(\beta_1-\beta_2)/(1+\nu)]}=-\left[\frac{\eta}{2}+\frac{\xi}{\rho^2}+2\delta+\sum_{n=1}^{\infty}\left\{\left[-\frac{(n+2-n^2)}{2\gamma^{2n+2}}\rho^n+\right.\right.\right.$$
$$\left.\left.\left.+\frac{(1-n)}{2}\left((n+1)\gamma^{-2n}-\omega\right)\rho^{n-2}+\frac{(n+1)}{2}\omega_1\rho^{-n-2}\right](a_{n1*}\sin n\theta+b_{n1*}\cos n\theta)\right\}\right];$$

$$\sigma'_{r\theta}=\frac{\sigma_{r\theta}(\rho,\theta)}{[E(\beta_1-\beta_2)/(1+\nu)]}=-\sum_{n=1}^{\infty}\left\{\left[\frac{(n^2+n)}{2\gamma^{2n+2}}\rho^n+\frac{(1-n)}{2}\left((n+1)\gamma^{-2n}-\omega\right)\rho^{n-2}-\frac{(n+1)}{2}\omega_1\rho^{-n-2}\right](a_{n1*}\cos n\theta-b_{n1*}\sin n\theta)\right\};$$

$$\sigma'_{\theta\theta}=\frac{\sigma_{\theta\theta}(\rho,\theta)}{[E(\beta_1-\beta_2)/(1+\nu)]}=-\left[\frac{\eta}{2}-\frac{\xi}{\rho^2}+2\delta+\sum_{n=1}^{\infty}\left\{\left[-\frac{(n+2)(n+1)}{2\gamma^{2n+2}}\rho^n-\right.\right.\right.$$
$$\left.\left.\left.-\frac{(1-n)}{2}\left((n+1)\gamma^{-2n}-\omega\right)\rho^{n-2}-\frac{(n+1)}{2}\omega_1\rho^{-n-2}\right](a_{n1*}\sin n\theta+b_{n1*}\cos n\theta)\right\}\right];$$

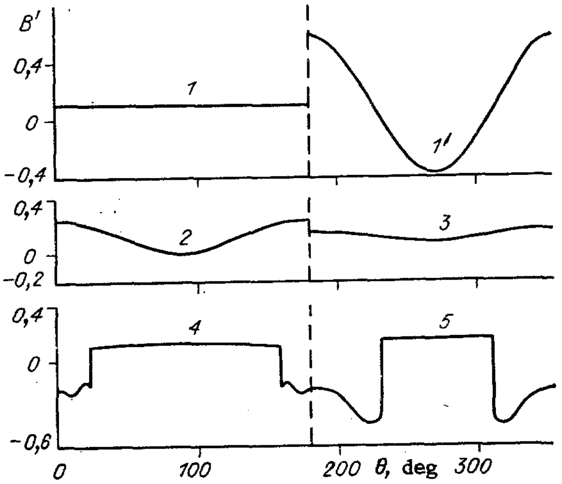

Fig. 3

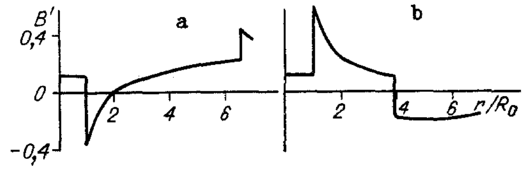

Fig. 4

$$\eta=\begin{cases}\dfrac{\beta_0}{\beta_1-\beta_2}, & 0\leqslant\rho<\gamma_0,\\ \dfrac{\beta_1}{\beta_1-\beta_2}, & \gamma_0<\rho<1+\zeta_{1*}(\theta),\\ \dfrac{\beta_2}{\beta_1-\beta_2}, & 1+\zeta_{1*}(\theta)<\rho<\gamma\,[1+\zeta_{2*}(\theta)];\end{cases}$$

$$\delta=-\frac{\beta_2}{4(\beta_1-\beta_2)}-\frac{\gamma_0^2(\beta_0-\beta_1)}{4\gamma^2(\beta_1-\beta_2)}-\frac{1}{4\gamma^2};$$

$$\sigma'_{xx}=\sigma'_{rr}\cos^2\theta-\sigma'_{r\theta}\sin 2\theta+\sigma'_{\theta\theta}\sin^2\theta;$$

$$\sigma'_{xy}=\frac{1}{2}(\sigma'_{rr}-\sigma'_{\theta\theta})\sin 2\theta+\sigma'_{r\theta}\cos 2\theta;$$

$$\sigma'_{yy}=\sigma'_{rr}\sin^2\theta+\sigma'_{r\theta}\sin 2\theta+\sigma'_{\theta\theta}\cos^2\theta.$$

The results of the solution of the problem are shown in Fig. 2. Dashed lines 1 and 2 show the initial (before heating and flow of the glass) configurations of boundaries Γ_1 and Γ_2, while solid lines 1' and 2' show the final configurations (1 and 2' – circles). Circle 3 corresponds to the boundary of the core. In the calculations, we modeled the plane cutting of the cladding sheath. It was assumed that $\gamma = 2$, $\gamma_0 = 0.2$, $\mu_1/\mu_2 = 0.2$, while the ratio of the interfacial force and surface tension $\alpha_1/\alpha_2 = 0$. We used 19 modes of the Fourier series. The calculated final form of the straining sheath, corresponding to the initial configuration of the cross section of the semifinished product shown in Fig. 2, is close to an ellipse. The values of coefficient $b_{21}*$, corresponding to different values of γ_0 with the remaining parameters of the problem fixed, were as follows: $\gamma_0 = 0.1, 0.2, 0.3, 0.4, 0.5$; $b_{21}* = -0.2823, -0.2776, -0.2683, -0.2523, -0.2260$. All of the values of γ_0 corresponded to a straining sheath having a final configuration in the form of an "ellipse." At small γ_0 (≤ 0.4), the form of the boundary Γ_1 is similar to the form obtained from the solution for a two-layer semifinished product [2, 3].

Figure 3 shows the distributions of dimensionless birefringence B' with a change in the polar angle θ in the plane of the cross section of the optical fiber (the graphs may be continued symmetrically across the boundary θ = 180°). Line 1 corresponds to $r = R_0 - 0$, 1' to $R_0 + 0$; 2 to $2R_0$; 3 to $3R_0$; 4 to $4R_0$; 5 to $5R_0$. In the last two cases, part of the circle on which B' is calculated lies within the cladding sheath. This is reflected in the discontinuities on curves 4 and 5. It was assumed in the calculations that $\beta_0 = \beta_2$. Thus, $(\beta_0 - \beta_1)/(\beta_1 - \beta_2) = -1$.

Figure 4 illustrates the change in B' in the cross section of the optical fiber in the radial direction [a) θ = 90°, b) θ = 0]. Birefringence in the cross section of a fiber similar to that being examined here (see Fig. 2) was calculated numerically in [9]. The results obtained in [9] are qualitatively close to the results of the analytical solution obtained in the present study (see Fig. 4). It should be noted that the discontinuities in the relations B'(r) in Fig. 4 are connected with crossing of the interfaces between the core and straining sheath and between the latter and the cladding sheath.

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STRUCTURE OF CHEMICALLY NONEQUILIBRIUM FLOWS WITH A SUDDEN CHANGE IN THE TEMPERATURE AND THE CATALYTIC PROPERTIES OF THE SURFACE

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The problem of chemically nonequilibrium flows in the neighborhood of a point where there is a sudden change in the temperature or the catalytic properties of the surface of a body is of undoubted interest from both theoretical and practical standpoints. Thus, the authors of [1-4] studied the effect of discontinuity of the catalytic properties of the surface on flow about the body within the framework of laminar boundary layer theory or the theory of hypersonic viscous shock layers. The problem was examined in [5-7] in a formulation which was the same except for the introduction of a hypothetical boundary layer immediately after the point of discontinuity: with the use of simplifying assumptions, the investigators succeeded in obtaining an analytic solution for the flow functions in the neighborhood behind the point of discontinuity of surface catalytic properties.

To describe the upstream propagation of disturbances from the point of discontinuity – such propagation being absent for boundary-value problems of the parabolic type [1-7] – the authors of [8-10] considered longitudinal diffusion in a certain region of the point; the substantiation for such a flow model for a removable discontinuity was presented in [11], where investigators made use of the method of combinable asymptotic expansions [12]. This method has already been used to solve many problems involving singular perturbations in fluid mechanics (see [13, 14] and their bibliographies, for example).

When analyzing the neighborhood of a point of discontinuity of surface catalytic properties, it is necessary to consider that in the transition from a noncatalytic surface to a surface which is ideally catalytic (for example), the density of the gas near the surface of the body is increased by a characteristic amount, i.e., the streamlines are shifted toward the surface of the body and the flow moves past a hypothetical depression. The main assumption of Prandtl's classical boundary-layer theory – that the longitudinal gradients of the flow functions are small compared to the transverse gradients – may be invalidated for such flows, and it becomes necessary to use the complete Navier–Stokes equations. A systematic analysis of the flow regimes around small two-dimensional irregularities on the surface of a body was performed in [15]. A solution to the problem of surface temperature

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